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# Viscoelastic Characterization of Solid Propellants by Transient Test Techniques

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## INTRODUCTION

The capability for performing an accurate engineering analysis of a structure is fundamentally dependent upon precise knowledge of the physical properties of its materials of construction. When the structure of interest is large or expensive, it is customary to proceed with an engineering analysis using laboratory material data to arrive at an evaluation of the structure's physical integrity under anticipated load. Since rocket motor grains must perform as structural members of a propulsion vehicle, as well as provide the source of propulsion, it is necessary that development of structural analysis capability for solid propellant rocket grains proceed apace with rocket development. The problem is a particularly acute one for motors that may contain hundreds of thousands of pounds of propellant. Construction of such large vehicles can only be begun after competent and reliable engineering analysis of the proposed structure.

About 90% of the material of construction of a solid rocket motor is propellant, which has physical properties that are sensitive functions of time and temperature. Unfortunately, there is no simple and reliable method whereby structural analyses of a propellant grain can be performed using conventional elastic methods employing time and temperature independent material property specification. However, many of the more important propellant grain structural analyses can be done using the methods of elasticity as modified by the principle of the viscoelastic analogy. A major problem connected with structural analysis using the viscoelastic analogy is a general lack of quantitative representation of propellant properties in a framework mathematically amenable to the desired solutions.

Research effort during the past years at the Grand Central Rocket Co. (GCR), has disposed of one of the main obstacles in structural analysis of propellant grains: viscoelastic linearity. Tests have shown that a polycarbutene propellant representative of many ballistically useful propellants is indeed a linear viscoelastic at practical stress and strain levels. Since the theoretical framework for both linear viscoelastic material representation and first approximation viscoelastic stress analysis has been established through the excellent work of Lee, Rivlin, Bland, and others, the basic propellant grain analysis problems are presently laboratory characterization of propellants and evaluation of the engineering utility of the theories.

The following paragraphs discuss the first of the problems: laboratory propellant characterization.

## DIFFERENTIAL OPERATOR MATERIAL PROPERTY SPECIFICATION

Many published solutions for stress distributions in linear viscoelastic bodies have been based on the linear differential operator specification of the relationship of time-dependent stress $\sigma(t)$ to the corresponding strain $\epsilon(t)$:

$$P\sigma(t) = Q\epsilon(t) \tag{1}$$

where

$$P = \sum_{i=0}^{n} a_i \frac{\partial^i}{\partial t^i} \tag{2}$$

and

$$Q = \sum_{i=0}^{m} b_i \frac{\partial^i}{\partial t^i} \tag{3}$$

in which $n$ and $m$ are not necessarily equal.[1,2] It is assumed in the definitions implicit to eqs. (1), (2), and (3), as well as throughout this paper, that only uniaxial stress in an incompressible material is considered. The constants $a_i$ and $b_i$ specify the stress-strain law for a material at a constant temperature. The order of $n$ with respect to $m$ and the presence or absence of the constant $a_0$ define various aspects of the limiting behavior of the material.[1] For example, when $a_0 = 0$, stress diminishes to zero at a constant strain as time increases.

The finite differential operator form of propellant viscoelastic property specification has been used in research at GCR in favor of, for example, integral property representation, because of its inherent simplicity and the availability of stress analysis solutions based on operators. One criticism of operator material specification that has been voiced is with regard to the complexity involved when the order of the operators is two or more. Contrary to the criticism, the mathematics of high order operators is straightforward and, most importantly, easily programmed for computer solution.

Routines for both operator coefficient determination and problem solving using twentieth order operators have been programmed. From a practical point of view, operators of order ten or less appear to be sufficiently accurate for problems involving time spans of $10^{-3}$ to $10^{7}$ sec.

## DETERMINATION OF OPERATOR COEFFICIENTS

The two basic experimental methods used to determine propellant viscoelastic characteristics are uniaxial creep and stress relaxation tests. The analysis for a stress relaxation test and the routine for determination of operator coefficients from the data are presented below.

Consider a bar of a linearly viscoelastic material as defined by the finite linear differential operator

$$\left[\sum_{i=0}^{n} a_i \frac{\partial^i}{\partial t^i}\right] \sigma(t) = \left[\sum_{i=0}^{n} b_i \frac{\partial^i}{\partial t^i}\right] \epsilon(t) \qquad (4)$$

which is strained uniaxially to a constant strain level $\epsilon(t_1)$ at time $t_1$, by a programmed strain input of the type

$$\epsilon(t)\Big|_{0<t<t_1} = \frac{A^*(1-\cos\gamma t)}{\gamma^2} \qquad (5)$$

$$\epsilon(t_1) = \frac{A^*}{\gamma^2} \qquad (6)$$

Taking the Laplace transform of eq. (4) for the input strain conditions as noted yields

$$\frac{\sigma(S)}{A^*} = \left[\frac{b_n S^n + b_{n-1} S^{n-1} + \ldots + b_1 S + b_0}{a_n S^n + a_{n-1} S^{n-1} + \ldots + a_1 S + a_0}\right] \times \left[\frac{1 + e^{-t_1 S}}{S(S^2+\gamma^2)}\right] \qquad (7)$$

Decomposition of the polynomial quotient in eq. (7) by partial fractions leads to

$$\frac{\sigma(S)}{A^*} = \left[\frac{A_{n+1}}{S} + \frac{A_1}{S+\alpha_1} + \ldots + \frac{A_n}{S+\alpha_n}\right] \times \left[\frac{1+e^{-t_1 S}}{S^2+\gamma^2}\right] \qquad (8)$$

where $\alpha_i$ are the roots of the polynomial denominator of eq. (7) and the $A_i$ are the partial fraction residuals. This algebraic rearrangement leads conveniently to the Laplace inversion to obtain the time plane solution

$$\begin{aligned} \frac{\sigma(t)}{A^*} &= \frac{A_{n+1}(1-\cos\gamma t)}{\gamma^2} \\ &+ \frac{A_{n+1}[1-\cos\gamma(t-t_1)]u(t-t_1)}{\gamma^2} \\ &+ \sum_{i=1}^{n} \frac{A_i}{\alpha_i^2+\gamma^2}\left[e^{-\alpha_i t} + \frac{\alpha_i}{\gamma}\sin\gamma t - \cos\gamma t\right] \\ &+ \sum_{i=1}^{n} \frac{A_i}{\alpha_i^2+\gamma^2} \\ &\times \left[e^{-\alpha_i(t-t_1)} + \frac{\alpha_i}{\gamma}\sin\gamma(t-t_1) - \cos\gamma(t-t_1)\right] \\ &\times u(t-t_1) \end{aligned} \qquad (9)$$

Experimental data for stress relaxation tests are fitted to eq. (9) to obtain the numerical values for the $A_i$ and $\alpha_i$. With known values for the coefficients in eq. (9), eq. (8) can be written directly in numerical form and the partial fraction form recomposed to obtain eq. (7). The coefficients of the polynomials in eq. (7) are thus the desired operator coefficients.

It is informative to note that the roots of the stress operator $\alpha_i$ can be interpreted directly as the reciprocal relaxation times of the material.

## MATERIAL CHARACTERIZATION IN THE MILLISECOND TIME DOMAIN

It is apparent from the analysis in the preceding section that precision control of test input conditions is necessary if accurate determinations of viscoelastic material characteristics are to be made. The problem is acute for propellants since most compositions display, at 70°F., relaxation times extending into the fractional millisecond range. Hence, controlled and analytic input conditions to a creep or stress relaxation test of a propellant are required. It is mechanically impractical to effect a finite strain or stress change in a propellant rapidly enough to consider it to be a step function and make use of the simplified analyses then possible.

In all propellant testing at GCR it has been found necessary to emphasize test input control in favor of test speed in creep and stress relaxation tests. Without control of the input conditions, test data could not be reduced with meaningful accuracy. While this procedure has yielded excellent propellant viscoelastic representations, the shorter relaxation times that dominate the behavior of the propellant during the first 50 to 100 msec. of its response could not be effectively isolated. The fundamental limitation results from the necessity for obtaining curve fit data in the first few milliseconds after termination of the input program to a higher degree of accuracy than is experimentally feasible. The solution to this problem is to impose rapid precision test input conditions on a sample. In general, what is desired is an analytical stress or strain input condition to a test that is imposed in 1 to 5 msec. with a high degree of accuracy. This speed requirement, while fast enough to obtain the data needed, is slow enough that inertial loading effects can be ignored, depending somewhat upon the precise strain-time function employed.

An analytically appealing strain input condition is a ramp strain approach at a constant strain rate to a constant strain level for examination of stress relaxation behavior. Indeed, this type of strain input condition has been used quite successfully, but only when the ramp times are quite long (>100 msec.). At higher ramp speeds, maintenance of a sufficiently linear ramp results in very high accelerations at the inception and termination of the ramp, causing inertial loading effects that are difficult to measure and interpret. Hence, effort has been turned toward generation of input functions through a servo-controlled high speed apparatus to obtain input functions such as a haversine or exponential strain (stress) approach to a constant strain (stress) level.

The haversine input condition is a mechanically as well as analytically desirable stress or strain input condition for two reasons. First, in the laboratory the force demands on a servomechanism are minimized and, second, direct "analog" solutions for strains in a grain become possible since the haversine input form for stress is an excellent approximation to pressure-time conditions during rocket motor ignition. Presently, development of an apparatus to provide the desired control parameters on a stress or strain program is being approached through modification of the GCR Plastechon tester. The design of modifications to the Plastechon servo and control units is presently being done under the sponsorship of the Plas-Tech Equipment Co.

## DISCUSSION

The GCR program of viscoelastic research is presently directed at a single objective: development of the capability for conducting engineering stress analyses of propellant grains. In essence, most of the theoretical aspects of this work have been resolved through the efforts of other workers in the field, notably Lee, of Brown University. Hence, GCR has been engaged in proof of principle and reduction to practice activities. The approach used in this work has been a pragmatic one. Thus far the progress has been excellent. Propellant viscoelastic linearity has been demonstrated and the data obtained have been used successfully in predicting propellant grain behavior.[3] Experimental demonstration of this latter point in large grains has been done. All property determination and grain behavior predictions have been done for a single, constant temperature. One fundamental guide line for the program has been that the problem will be assaulted and, for engineering purposes, solved at a single temperature before progressing to the study of temperature effects. Indeed, before use can be made of the time-temperature shift theory, it is necessary that its validity be experimentally established for the high accuracy level required in viscoelastic propellant characterization. Study of this aspect of propellant viscoelastic behavior has begun. Clearly, precision verification of a time-temperature shift relationship for propellants will greatly simplify experimental propellant property determinations.

In conclusion, it is important to emphasize the salient points examined above. Precision determination of propellant viscoelastic properties is a vital and absolutely essential step in propellant grain analysis. Approximation analytical and test methods of the type commonly considered in connection with elastic analysis are not useful in propellant grain studies. The theoretical framework for viscoelastic stress analysis exists; preliminary demonstrations of its validity have been done. A vital lack exists in laboratory experimental techniques directed at support of this activity in connection with rocket motor grain analysis. The lack is a technically apparent one, however, and rapid progress toward remedy of the deficiency is being made.

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## References

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## Synopsis

The experimental determination of solid propellant viscoelastic physical characteristics using specialized test procedures is described. Experimental and analytical difficulties connected with propellant physical characterization in the millisecond time domain are discussed. Viscoelastic property specification by linear differential operator techniques is described.

## Résumé

On décrit la détermination expérimentale des caractéristiques physiques viscoélastiques de mélanges propulsants solides utilisant des méthodes de tests spécialisées. Les difficultés expérimentales et analytiques liées à la caractérisation physique du propulseur dans le domaine de la milliseconde sont discutées. On décrit la spécification d'une propriété viscoélastique par des techniques d'opérateur différentiel linéaire.

## Zusammenfassung

Die experimentelle Bestimmung viscoelastischer physikalischer Charakteristika von festen Raketentreibstoffen mit speziellen Testverfahren wird beschrieben. Experimentelle und analytische Schwierigkeiten bei der physikalischen Charakterisierung von Treibstoffen im Millisekundenbereich werden diskutiert. Die Spezifizierung viskoelastischer Eigenschaften durch die Anwendung linearer Differentialoperatoren wird beschrieben.

## Discussion

**Chairman Eirich:** In view of your successes, do you consider them proof that these materials do behave as linear viscoelastic materials? Do you consider the theoretical fit in your last analysis of the curves a proof that one may look upon the rocket grain as a linear viscoelastic body?

**Answer:** Yes, I think one can say that the propellant, for engineering purposes, unquestionably indicates linear behavior.

**Chairman Eirich:** You have shown the slump in a propellant grain. Can you define exactly what you understand concerning this slump; my opinion is that it is the differential change in the parameters of the material. Further, this slump is not measured over an appreciable time scale. When your other equation is applied, in which the acceleration comes in at the moment the grain is being moved, then of course your time scale becomes even shorter. Do you think that under these circumstances it still acts as a linear viscoelastic body?

**Answer:** I think it would probably be a better linear viscoelastic material under rapid straining. The data I used in the analysis were obtained in uniaxial tests; they were used in analyzing structure in a three-dimensional stress state.

Under such conditions I do not think that one may, on the basis of a first analysis, unequivocally state that this is proof of linearity in the propellant. The proof of linearity, however, is defined as one conducts a stress relaxation test and obtains a particular set of operator coefficients, then conducts a creep test, analyzes the data, and obtains a second set of operator coefficients. Comparing the first set of operator coefficients with the second set, one observes they are the same and thus proves that it is a linear material.